FURTHER REMARKS ON THE SURFACE *VIS IMPRESSA* CAUSED BY A FLUID-SOLID CONTACT

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ABSTRACT

It is well-known that, nano-mechanics should take into account not only physical phenomena occuring within the bulk but, first of all, the physical phenomena appropriate for a surface of two materials contact. The huge volume density of internal surfaces as well countours lines located within the nanomaterial results in our interest in, apart from classical form of mass, momentum and entropy transport, those modes of transportation where a carrier of physical property follows a free path having of a dimension greater than nanostructure characteristic dimension. The mode of transport dominated by mechanical, thermal and electrical slip of carried bounding off walls (a surface of separation) is called usually in physics "a ballistic mode". In the paper the appropriate Newtonian surface *vis impressa* responsible for the ballistic mode of transport is defined, classified and explained. We postulate that generally surface *vis impressa* can be additivelly splited onto friction and mobility forces.

MOVING SHELL-LIKE CONTACT

We assume that the fluid-solid contact layer (denoted as $\mathcal{M}^+\mathcal{M}^-$) can be treated as thin domain moving in a space with a geometrical, migration velocity $\mathbf{w}$. This shell-like domain divides the continuum into a continuum $\mathcal{A}$ - that is a fluid under consideration, and a continuum $\mathcal{B}$ which can be a free surface, solid body or second fluid, as in Fig. 1. If both $\mathcal{A}$ and $\mathcal{B}$ are fluids then it is the fluid-solid contact layer represents the moving interfacial region, where physical properties change in a radical manner. For instance in a thin transition layer between liquid and vapor, the change of density is so noticeable, that it looks like a jump throughout the layer thickness. Therefore, we assume that in the layer we observe so-called „apparent" material properties, quite different than in bulk continuum $\mathcal{A}$ and $\mathcal{B}$. Thus we define an excess of layer density ρ_s [kg m^{-2}], the layer particle velocity $\mathbf{v}_s$ [m s^{-1}], an excess of layer momentum density $\rho_s\mathbf{v}_s$, and a surface excess of momentum flux $\mathbf{p}_s$, [1; 6; 7; 20].

In general, this layer moves with the geometrical velocity $\mathbf{w}$ that differs from material velocity $\mathbf{v}_A$ in $\mathcal{A}$, velocity $\mathbf{v}_B$ in $\mathcal{B}$, and velocity $\mathbf{v}_s$ in $\mathcal{M}^+\mathcal{M}^-$. In particular case, the velocity $\mathbf{w}$ denotes the rate of changing a phase transition surface within the fluid being at rest. Usually, the component w_n normal to moving middle surface $\mathcal{M}$, differs from normal components of $\mathbf{v}_A$, $\mathbf{v}_B$ and $\mathbf{v}_s$. It practically means that there is also a mass transport across the layer. Indeed, the geometrical velocity field is not *a priori* known, and can be determined from a special evolution equation, [1; 19]. If $\mathbf{w} = \mathbf{v}_s$ then the moving layer is material, if $\mathbf{w} = \mathbf{v}_s\mathbf{I}_s + w_n\mathbf{n}$ the surface is semi-coherent (Fig. 1). Navier and Stokes have assumed, that the surface layer density is equal to zero. Apparently, we want to determine the slip velocity $\mathbf{v}_s$ from an independent balance of the layer momentum. In special cases however, it simplifies to the well-known Cauchy balance of the boundary traction forces. For immiscible liquids being in contact, the tangential components $\mathbf{v}_s\mathbf{I}_s$ can be approximately described to be $\frac{1}{2}(\mathbf{v}_A + \mathbf{v}_B)\mathbf{I}_s$. Quite similarly,

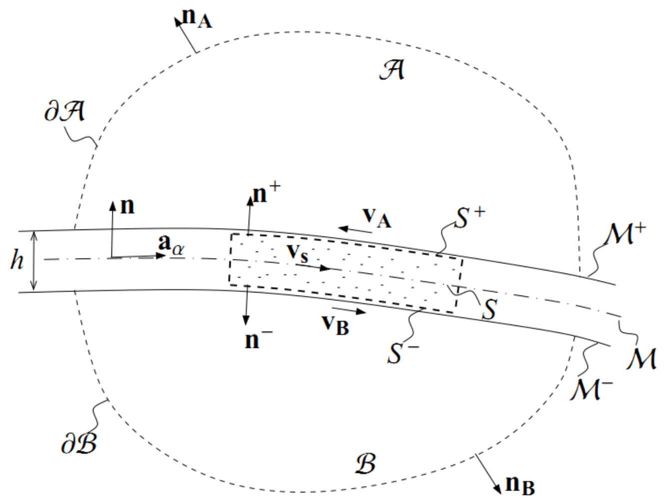

Figure 1. Outline of the fluid-soild contact layer

only in a special case is $\rho_s = \frac{1}{2}(\rho_A + \rho_B)h$, where h is a finite thickness of the layer[1].

We introduce a new concept of an „excess of momentum flux" within the fluid-solid contact layer, which is described by a surface symmetrical diade $\mathbf{p}_s$. It governs the momentum transport within the layer, and therefore it has a tangential and normal components. We postulate the surface momentum flux in a following form:

$$\mathbf{p}_s(\xi) = p^{\alpha\beta}\mathbf{a}_\alpha \otimes \mathbf{a}_\beta + p^{n\alpha}\mathbf{n} \otimes \mathbf{a}_\alpha + p^{\alpha n}\mathbf{a}_\alpha \otimes \mathbf{n} + p^{nn}\mathbf{n} \otimes \mathbf{n}\,, \tag{1}$$

where ξ^α, $\alpha = 1, 2$ are a local surface curvilinear coordinates

[1] We are based on a general surface kinematics elaborated by [20]. The general form of the surface balances of mass, momentum, angular momentum, energy, entropy, etc. is given by [16; 9; 21].

on $\mathcal{M}$, and $\mathbf{a}_\alpha$, $\mathbf{n}$ ($\alpha = 1, 2$) are the base vectors on the middle surface of the layer $\mathcal{M}$. Since the physical properties of the layer are unknown *a priori*, they depend on the resulting apparent properties in both continua $\mathcal{A}$ and $\mathcal{B}$. For example, elastic recoverable properties of $\mathbf{p}_s$ depend on an actual shape of the surface $\mathcal{M}$. Many authors postulate, that due to strong induced elasticity of the fluid layer, it changes from the elastic fluid (only recoverable spherical deformations) into an elastic fluid with recoverable shape deformations [7]. Similarly, owing to induced strong anisotropy, the internal viscosity of the fluid layer can be described by four apparent viscosity coefficients, [3; 11].

Let us now recall a few mathematical relations required for establishing of balance of the layer mass and momentum. At first the Weatherburn surface fundamental diades can be introduced, [17]:

$$\mathbf{I}_s = \mathbf{I} - \mathbf{n} \otimes \mathbf{n} = \mathrm{grad}_s \mathbf{x}_s = a^{\alpha\beta} \mathbf{a}_\alpha \otimes \mathbf{a}_\beta \,, \tag{2}$$

$$\mathbf{II}_s = -\mathrm{grad}_s \mathbf{n} = b^{\alpha\beta} \mathbf{a}_\alpha \otimes \mathbf{a}_\beta \,, \tag{3}$$

which are called the first and second fundamental form of the surface $\mathcal{M}$. As far as the surface gradient acts also on the coordinate dependent base $\mathbf{a}_\alpha$, $\mathbf{n}$, then the surface gradient of velocity is calculated to be:

$$\begin{aligned} \mathrm{grad}_s \mathbf{v}_s &= (v^\alpha \mathbf{a}_\alpha + v_n \mathbf{n}) \otimes \nabla_\beta \mathbf{a}^\beta \\ &= \left(v_{\alpha|\beta} - v_n b_{\alpha\beta}\right) \mathbf{a}^\alpha \otimes \mathbf{a}^\beta \\ &\quad + (v^\alpha b_{\alpha\beta} + v_{n,\beta}) \mathbf{n} \otimes \mathbf{a}^\beta, \end{aligned} \tag{4}$$

and the surface divergence of velocity vector is based on the contraction $C_{1,2}$:

$$\begin{aligned} \mathrm{div}_s \mathbf{v}_s &= C_{1,2} \mathrm{grad}_s \mathbf{v}_s {=} (v_{\alpha|\beta} - v_n b_{\alpha\beta}) a^{\alpha\beta} \\ &= v^\alpha{}_{|\alpha} - v_n b^\alpha{}_\alpha = \mathrm{div}_s \left(\mathbf{v}_{s\|}\right) - v_n \mathrm{I}_b. \end{aligned} \tag{5}$$

where the invariants of the second fundamental form of the curvature diade are: $\mathrm{I}_b = \mathrm{tr}\mathbf{II}_s = b^\alpha{}_\alpha = b^1{}_1 + b^2{}_2 = \left(\frac{1}{r_1} + \frac{1}{r_2}\right)$, $\mathrm{II}_b = \det \mathbf{II}_s = \det(b_{\alpha\beta})$ and $C_{1,2}$ denotes contraction of first and second base. In analogy to the three-dimensional case, the rate of surface deformation is defined as a symmetric part of the surface gradient of velocity:

$$\begin{aligned} \mathbf{d}_s &= \frac{1}{2}\left(\mathrm{grad}_s \mathbf{v}_s + \mathrm{grad}_s^T \mathbf{v}_s\right) \\ &= \left[\frac{1}{2}\left(v_{\alpha|\beta} + v_{\beta|\alpha}\right) - v_n b_{\alpha\beta}\right] \mathbf{a}^\alpha \otimes \mathbf{a}^\beta \\ &\quad + \frac{1}{2}\left(v^\alpha b_{\alpha\beta} + v_{n,\beta}\right)\left(\mathbf{n} \otimes \mathbf{a}^\beta + \mathbf{a}^\beta \otimes \mathbf{n}\right). \end{aligned} \tag{6}$$

The first invariant of $\mathbf{d}_s$ is in analogy to 3D:

$$\mathrm{I}_{\mathbf{d}_s} = \mathrm{tr}\mathbf{d}_s = C_{1,2} \mathbf{d}_s {=} v^\alpha{}_{|\alpha} - v_n \mathrm{I}_b \,. \tag{7}$$

Similarly, the surface gradient of the flux of momentum is:

$$\begin{aligned} \mathrm{grad}_s \mathbf{p}_s &= \mathbf{p}_s \otimes (\nabla_\gamma \mathbf{a}^\gamma) = p^{\alpha\beta}{}_{|\gamma} \mathbf{a}_\alpha \otimes \mathbf{a}_\beta \otimes \mathbf{a}^\gamma \\ &+ p^{\alpha\beta} b_{\alpha\gamma} \mathbf{n} \otimes \mathbf{a}_\beta \otimes \mathbf{a}^\gamma + p^{\alpha\beta} b_{\beta\gamma} \mathbf{a}_\alpha \otimes \mathbf{n} \otimes \mathbf{a}^\gamma \\ &+ p^{n\alpha}{}_{|\gamma} \left(\mathbf{n} \otimes \mathbf{a}_\alpha \otimes \mathbf{a}^\gamma + \mathbf{a}_\alpha \otimes \mathbf{n} \otimes \mathbf{a}^\gamma\right) \\ &+ \left(2p^{n\alpha} b_{\alpha\gamma} + p^{nn}{}_{|\gamma}\right) \mathbf{n} \otimes \mathbf{n} \otimes \mathbf{a}^\gamma \\ &- p^{n\alpha} b^\varepsilon{}_\gamma \left(\mathbf{a}_\varepsilon \otimes \mathbf{a}_\alpha \otimes \mathbf{a}^\gamma + \mathbf{a}_\alpha \otimes \mathbf{a}_\varepsilon \otimes \mathbf{a}^\gamma\right) \\ &- p^{nn} b^\varepsilon{}_\gamma \left(\mathbf{a}_\varepsilon \otimes \mathbf{n} \otimes \mathbf{a}^\gamma + \mathbf{n} \otimes \mathbf{a}_\varepsilon \otimes \mathbf{a}^\gamma\right), \end{aligned} \tag{8}$$

and its divergence:

$$\begin{aligned} \mathrm{div}_s \mathbf{p}_s &= C_{2,3} \mathrm{grad}_s \mathbf{p}_s \\ &= \left(p^{\alpha\beta}{}_{|\beta} - p^{n\beta} b^\alpha{}_\beta - \mathrm{I}_b p^{\alpha n}\right) \mathbf{a}_\alpha \\ &+ \left(p^{\alpha\beta} b_{\alpha\beta} + p^{n\alpha}{}_{|\alpha} - \mathrm{I}_b p^{nn}\right) \mathbf{n}. \end{aligned} \tag{9}$$

where $C_{2,3}$ means scalar multiplication second & third vector of base (operation of contraction $C_{2,3}$).

MOMENTUM BALANCES WITHIN A CONTACT THIN LAYER

The local form of the momentum balance can be finally written as[2] [2]:

$$\partial_t (\rho \mathbf{v}) + \mathrm{div}(\rho \mathbf{v} \otimes \mathbf{v} + \mathbf{p}) = \rho \mathbf{b} \quad \text{for} \quad \mathcal{A} \cup \mathcal{B}, \tag{10}$$

$$\begin{aligned} &\partial_t (\rho_s \mathbf{v}_s) + \mathrm{div}_s \left(\rho_s \mathbf{v}_s \otimes \mathbf{v}_{s\|}\right) - w_n \mathrm{I}_b \rho_s \mathbf{v}_s + \mathrm{div}_s \mathbf{p}_s \\ &+ \partial_n (\mathbf{p}_s \mathbf{n}) + [\mathbf{p}_A \mathbf{n}_A + \mathbf{p}_B \mathbf{n}_B + \mathbf{f}_{SA} + \mathbf{f}_{SB}] = \rho_s \mathbf{b}_s \\ &+ \dot{m}_A (\mathbf{v}_A - \mathbf{v}_s) + \dot{m}_B (\mathbf{v}_B - \mathbf{v}_s) \quad \text{on} \quad \mathcal{M}. \end{aligned} \tag{11}$$

Repeating now the reasoning of d'Alembert and Euler, we can define a surface d'Alembert-Euler acceleration vector to be:

$$\mathbf{a}_s = \frac{d_s}{dt} \mathbf{v}_s = \partial_t \mathbf{v}_s + (\mathrm{grad}_s \mathbf{v}_s) \mathbf{v}_{s\|} \,. \tag{12}$$

Employing the surface identity, instead of divergence of the convective flux of surface momentum we obtain:

$$\rho_s \mathbf{a}_s = \partial_t (\rho_s \mathbf{v}_s) + \mathrm{div}_s \left(\rho_s \mathbf{v}_s \otimes \mathbf{v}_{s\|}\right) \tag{13}$$

The fluid-solid contact layer in generalized form is describied now by the layer balances of mass and momentum. These are two additional nonlinear differential equations for two additional fields of unknowns, i.e. the surface mass density ρ_s and the layer slip velocity $\mathbf{v}_s$. These equations are both geometrically and physically nonlinear, and should be solved using any discretization method (FEM, FVM), under assumption that the surface $\mathcal{M}$ possesses an independent from the bulk space discretization. In the case when $\mathcal{M}^-$ is a fixed solid surface, the geometrical velocity $\mathbf{w} = 0$, and then discretization mesh could be fixed in the marching time of numerical solution. Apparently, if $\mathbf{w} \neq 0$, then a moving, self deforming mesh should be resolved together with surface mass and surface momentum equations, and the appropriate set of equations for bulk. There are

[2] An example how to define $\mathbf{p}_B$ for the deformable wall is given in the paper by dell'Isola et al. [8], eq.(40)

different cases of using the Navier-Stokes layer balances in the literature. For instance, when $\mathcal{A}$ and $\mathcal{B}$ are ideal, non-viscous Euler fluids, and the surface density is equal to zero $\rho_s = 0$, and the layer momentum flux is omitted $\mathbf{p}_s = 0$, then the surface mass and momentum equations reduce to the generalized form of the Rankine-Hugoniot jump conditions:

$$\begin{cases} \dot{m}_A = \dot{m}_B \\ \dot{m}_A \mathbf{v}_A + p_A \mathbf{n}_A = \dot{m}_B \mathbf{v}_B + p_B \mathbf{n}_B \end{cases}, \tag{14}$$

where p_A, p_B are thermodynamic pressure in the Euler fluids $\mathcal{A}$ and $\mathcal{B}$, respectively. If, additionally $\mathbf{w} = 0$, and there is an additional contribution to the surface diade $\mathbf{p}_s = \gamma \mathbf{I}_s$, then the layer momentum balance leads to the generalized Young-Laplace equation:

$$\begin{aligned} &\mathrm{div}_s\left(\gamma \mathbf{I}_s\right) + p_A \mathbf{n}_A + p_B \mathbf{n}_B \\ &= \left[\gamma\left(\frac{1}{r_1} + \frac{1}{r_2}\right) + p_A - p_B\right]\mathbf{n} = 0\,. \end{aligned} \tag{15}$$

If an interfacial density is omitted i.e. $\rho_s = 0$, the difference between the external friction forces $\mathbf{f}_{SA}$ and $\mathbf{f}_{SB}$ simply vanishes then, and a single layer friction force exists:

$$\mathbf{f}_{AB} = \mathbf{f}_{SA} + \mathbf{f}_{SB} = \nu\left(\mathbf{v}_A - \mathbf{v}_B\right), \tag{16}$$

where ν is an external viscosity coefficient. It is an exact form of an external friction force proposed by Navier ($\mathbf{v}_B = 0$) and Stokes ($\mathbf{v}_B = \mathbf{v}_{wall}$). Assuming, that the continuum $\mathcal{A}$ is an incompressible viscous fluid: $\mathbf{p}_A = p\mathbf{I} - 2\mu\mathbf{d}$, and the continuum $\mathcal{B}$ is a rigid, fixed solid body: $\mathbf{p}_B = 0$, $\mathbf{v}_B = 0$, we obtain the Navier slip boundary condition:

$$\mathbf{f}_{AB} + \mathbf{p}_A \mathbf{n}_A = \nu \mathbf{v}_A + \left(p\mathbf{I} - 2\mu\mathbf{d}\right)\mathbf{n} = 0 \quad \text{on} \quad \mathcal{M}, \tag{17}$$

where $\mathbf{v}_s = \mathbf{v}_{A|\mathcal{M}}$ is identified with the slip velocity.

Let note that the layer flux of momentum is responsible for recoverable and viscous transport: $\mathbf{p}_s = \mathbf{p}_s{}^{(c)} + \mathbf{p}_s{}^{(\nu)}$. The first most important part of the elastic recoverable diade $\mathbf{p}_s^{(c)}$, that is known as the capillarity diade, can be described by the surface tension γ. This quantity was introduced to the process of mathematical modeling by Young, Laplace and Poisson. The second contribution comes from the recoverable stresses called the surface bending C_1, C_2, introduced by Gibbs. There is also a layer „normal pressure" ϖ, introduced by Stokes. These altogether lead to the following definition of the capillarity diade:

$$\mathbf{p}_s{}^{(c)} = \varpi \mathbf{n} \otimes \mathbf{n} + \gamma \mathbf{I}_s + C\mathbf{II}_s, \quad \partial_n\left(\mathbf{p}_s \mathbf{n}\right) = \varpi \mathbf{n}, \tag{18}$$

where $2C = C_1 + C_2$, and $\mathrm{div}_s \mathbf{p}_s{}^{(c)} = \gamma \mathrm{I}_b \mathbf{n} + C\left(\mathrm{I}_b^2 - 2\mathrm{II}_b\right)\mathbf{n}$. A quite general form of the capillarity diade has been proposed recently [1] as:

$$\mathbf{p}_s{}^{(c)} = \boldsymbol{\gamma}_0 - \mathbf{II}_s \boldsymbol{\gamma}_1 + \mathbf{n} \otimes \mathbf{I}_s \mathrm{div}_s\left(\boldsymbol{\gamma}_1 - \mathbf{II}_s \boldsymbol{\gamma}_2\right), \tag{19}$$

where the surface capillary measures can be defined to be spherical:

$$\boldsymbol{\gamma}_0 = \gamma \mathbf{I}_s, \quad \boldsymbol{\gamma}_1 = C\mathbf{II}_s, \quad \boldsymbol{\gamma}_2 = K\mathbf{III}_s\,. \tag{20}$$

These capillary measures are expressed in terms of the first, second and third fundamental surface forms, and γ, C, K are the surface tension, bending and torque, respectively.

The viscous properties of the Navier-Stokes layer depend on the so-called „apparent viscosity" which, in general, possesses a transversal anisotropy, [11]. One can define the viscous surface stresses by using the surface diade of the rate of deformation and a normal change $v_{n,n}$:

$$\begin{aligned} \mathbf{p}_s{}^{(\nu)} &= \lambda'\left(\mathrm{tr}\mathbf{d}_s\right)\mathbf{I}_s + \lambda'' v_{n,n} \mathbf{n} \otimes \mathbf{n} \\ &+ 2\mu' \mathbf{I}_s \mathbf{d}_s \mathbf{I}_s + 2\mu''\left(\mathbf{d}_s - \mathbf{I}_s \mathbf{d}_s \mathbf{I}_s\right). \end{aligned} \tag{21}$$

This diade does not undergo the classical 3D de Saint-Venant condition, saying that the viscous stresses must be traceless. For a special case when $\lambda'' = \mu'' = 0$, this constitutive relation was proposed by B.M.J. Boussinesq (1913), [4; 19]:

$$\mathbf{p}_s{}^{(\nu)} = \left(\lambda' - \mu'\right)\left(\mathrm{tr}\mathbf{d}_s\right)\mathbf{I}_s + 2\mu' \mathbf{I}_s \mathbf{d}_s \mathbf{I}_s\,. \tag{22}$$

The formula for surface viscosity coefficients λ', μ' needs extended investigations.

SURFACE FRICTION *VIS IMPRESSA* CLASSIFICATION

Let us consider now a more consistent velocity slip boundary conditions that should be consistent with the Newton postulate stating, that a friction phenomenon depends on three components: the pressure dependent part, the relative velocity part, and the square velocity dependent part. Let the Newton postulate be true for a fluid in the bulk as well as for the thin layer on a boundary surface realizing a contact with a solid surface. Then taking into account, we have more consistent definition of the surface friction force:

$$\begin{aligned} \mathbf{f}^f_{AB} &= f_{SS'} N \frac{\mathbf{v} - \mathbf{v}_{wall}}{\left|\mathbf{v} - \mathbf{v}_{wall}\right|} + \nu\left(\mathbf{v} - \mathbf{v}_{wall}\right) \\ &+ f_\kappa\left(\mathbf{v} - \mathbf{v}_{wall}\right)^2 \frac{\mathbf{v} - \mathbf{v}_{wall}}{\left|\mathbf{v} - \mathbf{v}_{wall}\right|}\,. \end{aligned} \tag{23}$$

where $f_{SS'}$, ν, f_κ are cohesive, external friction and kinematic friction coefficients and $N = \mathbf{n} \cdot \left(\mathbf{p}_A - \mathbf{p}_B\right)\mathbf{n}$ is contact normal force. Some consistencies of this condition can be simply recognized if we compare the internal and external coefficients that appear in the model. This consistency can even be extended on reversible properties of the model i.e. the internal (Euler) and the external (Stokes) pressures p and ϖ, respectively. In the Table1 the comparison of these properties is shown .

The better consistency of the above model results from the fact that it needs three coefficients of internal friction (k_{vis}, μ_1, μ_2) and three coefficients of external friction ($f_{SS'}$, ν, f_k), respectively. Therefore, we can define a ratio between the internal and external friction by a dimensionless coefficient λ_{vis}, and two lengths of velocity slip: $l_{1\nu}$ and $l_{2\nu}$ (see: table 1). Having a measure of internal properties of friction, one can connect the external properties of friction at the fluid-solid contact layer by appropriate closures written for λ_{vis} , $l_{1\nu}$ and $l_{2\nu}$, respectively.

Table 1. Comparison of a concise model of internal and external friction, according to Newton's postulate. The model (†) of a viscous bulk pressure has been proposed by Natanson [15].

	Internal (bulk)	External (boundary)	Characteristic ratio
Elastic pressure	p [Nm^{-2}]	ϖ [Nm^{-2}]	$\lambda_{\text{press}} = p/\varpi$
frictional pressure	$p_{\text{vis}} = k_{\text{vis}} J^\dagger$ [Pa]	$f_{SS'}$	$\lambda_{\text{vis}} = k_{\text{vis}}/f_{SS'}N$
linear slip velocity	μ_1 [Nsm^{-2}]	ν [Nsm^{-3}]	$l_{1\nu} = \mu_1/\nu$
square slip velocity	μ_2 [$\mathrm{Ns^2m}^{-2}$]	f_k [$\mathrm{Ns^2m}^{-3}$]	$l_{2\nu} = \mu_2/f_k$

SURFACE MOBILITY *VIS IMPRESSA* CLASSIFICATION

Here, we must note that the previous literature statements of the phenomena of surface mobility, called transpiration, should be taken into account to the proper definition of surface friction. Yet another mobility force, other than the difference of pressure or temperature, was discovered by Graham in 1849. He found a new kind of transpiration called „atomisis"[18; 14]. This phenomena is nowadays called „diffusional transpiration" or „diffusionphoresis". It is quite different kind of flow than the classical transpiration flow induced by difference of the normal surface pressures, i.e. „pressure transpiration". The diffusion transpiration deals with a flow of gas mixture by a long capillary pipe, where there is another interaction of every mixture component with a surface. It leads to the mixture separation. In this case the most important is a coefficient of diffusion mobility c_{vN}. Another type of induced motion is due to the difference of an electric potential ϕ on a surface. This phenomenon is called „electrophoresis" and is governed by an electro-mobility coefficient[3] $c_{v\phi}$. Other mobility mechanism is connected with the phase transition change, [2] and the surface gradient of the phase order parameter x.

Let us note that these all types of mobility, i.e., pressure, thermal, diffusional, phase, and electrical define only an external mobility force in the fluid-solid contact layer. This force, partially given by Reynolds [18] and Maxwell [14], can be generalized to:

$$\begin{aligned} \mathbf{f}^m_{AB} &= -c_{v\varpi}\mathrm{grad}_s\varpi - c_{v\theta}\mathrm{grad}_s\theta \\ &-c_{vN}\mathrm{grad}_s N - c_{v\phi}\mathrm{grad}_s\phi - c_{vx}\mathrm{grad}_s x \ , \end{aligned} \tag{24}$$

where $c_{v\theta}$ - the thermo-mobility coefficient, c_{vN} - the concentration-mobility coefficient, $c_{v\phi}$ - electro-mobility coefficient, $c_{v\varpi}$ - the pressure-mobility coefficient, c_{vx} - the phase mobility coefficient.

In a special case, when gas is at rest, we can observe a motion of the particle induced by different surface *vis impressa*. This kind of motion is called in the literature the „phoretic motion" [5]. In general, any nano-particle immersed in the fluid may undergo simultaneously five types of motions which are shown in Table 2[4].

Table 2. Five kinds of motions connected with the surface mobility of a particle immersed in a fluid at rest. Here: $c_{v\theta}$ - the thermo-mobility coefficient, c_{vN} - the concentration-mobility coefficient, $c_{v\phi}$ - electro-mobility coefficient, $c_{v\varpi}$ - the pressure-mobility coefficient, c_{vx} - the phase mobility coefficient.

Phenomena	Corresponding velocity	Driving potential
thermophoresis	$\mathbf{v}_{wall} = c_{v\theta}\mathrm{grad}_s\theta$	temperature θ
diffusionphoresis	$\mathbf{v}_{wall} = c_{vN}\mathrm{grad}_s N$	concentration N
electrophoresis	$\mathbf{v}_{wall} = c_{v\phi}\mathrm{grad}_s\phi$	electric potential ϕ
pressurephoresis	$\mathbf{v}_{wall} = c_{v\varpi}\mathrm{grad}_s\varpi$	pressure ϖ
phasephoresis	$\mathbf{v}_{wall} = c_{vx}\mathrm{grad}_s x$	order parameter x

COMBINED SURFACE FRICTION AND MOBILITY

Let postulate surface *vis impressa* to be:

$$\mathbf{f}_{AB} = \nu\left(\mathbf{v} - \mathbf{v}_{wall} - c_{v\theta}\mathrm{grad}_s\theta\right) . \tag{25}$$

The thermo-mobility coefficient $c_{v\theta}$ should be formulated, according to Maxwell's slip formula [14], as a coefficient that is not dependent on the property of the solid surface:

$$c_{v\theta} = \frac{3}{4}\frac{\mu}{\rho\theta} . \tag{26}$$

Equation (25) is called the „Maxwell slip boundary layer". Let us note that in this equation very particular role plays the gradient of temperature θ. It is a completely external surface effect which is not connected with any form of stress tensor. It means that the motion of the gas close to a solid surface, in general is governed by two kinds of forces. The first is a mechanical one, which is connected with the external viscosity, and the second one is a temperature gradient which drives of gas particle close to the surface from colder to hotter part. Therefore the coefficient of thermal mobility $c_{v\theta}$ is independent from mechanical layer properties and should be experimentally verified[5].

Finally, let us recall Maxwell solution for a flow of a gas in a long capillary tube having inner radius a, which occurs under

[3]Electrophoresis was discovered by von Smoluchowski in 1916 [22]. See also: H.J. Keh, J.L. Anderson, Boundary effects on electrophoretic motion of colloidal sphere, *J. Fluid Mech.* 153,417-439(1985)

[4]These phenomena must be distinguished from the motion-less phenomena like: „temperature jump", „concentration jump", „potential jump" related with the external heat conductivity, external mass diffusivity, and external electric conductivity coefficients, respectively. Recently the phenomenon of jump concentration of salt in a gel mixture has been discovered by [12].

[5]There are numerous modern papers that mention about the proper experiments. The impressive electrokinetic properties predicted for a carbon nanotube channels have not yet been measured in careful experiments, [10].

two kind of driving forces. These forces are a bulk pressure transpiration due to difference of pressure at the ends of the tube, and the surface thermal transpiration due to difference of temperature at the same ends of the tube. Since the gas is flowing from higher to lower pressure and, simultaneously, from the colder to the hotter end, then these effects can be summarized. In a particular case, where the driving forces are opposite and equal themselves, there is no net outflow of gas from the capillary. Then an enhancement of mass flux due to the Maxwell slip is[6]:

$$\frac{Q_{Maxwell}}{Q_{Poiseuille}} = \left(1 + 4\frac{l_s}{a}\right) - \frac{8}{\pi} c_{v\theta} \frac{\mu}{\rho a^4} \frac{d\theta}{dz} \left(\frac{dp}{dz}\right)^{-1} . \quad (27)$$

This enhancement is essential only if the inner radius a is small in comparison with the slip length l_s and thermal mobility $c_{v\theta}$ is small. Thermal contribution to the slip is important when the gas is rarefied. Both driving forces (per unit of length of the pipe): dp and $d\theta$, can be in opposition. In a particular case there is no flow in the pipe $Q = 0$. Then we have[7]:

$$\frac{dp}{d\theta} = 6\frac{\mu^2}{\rho\theta} \frac{1}{a^2 + 4l_s a} . \quad (28)$$

For given temperature difference $d\theta = 100$ K, under the pressure of 40 mm of mercury, and assuming $l_s = 0.00016$ cm, this formula leads to the resulting pressure at the hot end which exceed that at the cold end by about 1.2 millionth of the atmosphere. Modern numerical techniques allowed us to reconstruct this experiment by means of Finite Volume Method. Obtained results are however slightly different - see Fig. 2, b) for which $\dot{\mathrm{m}} = 0$.

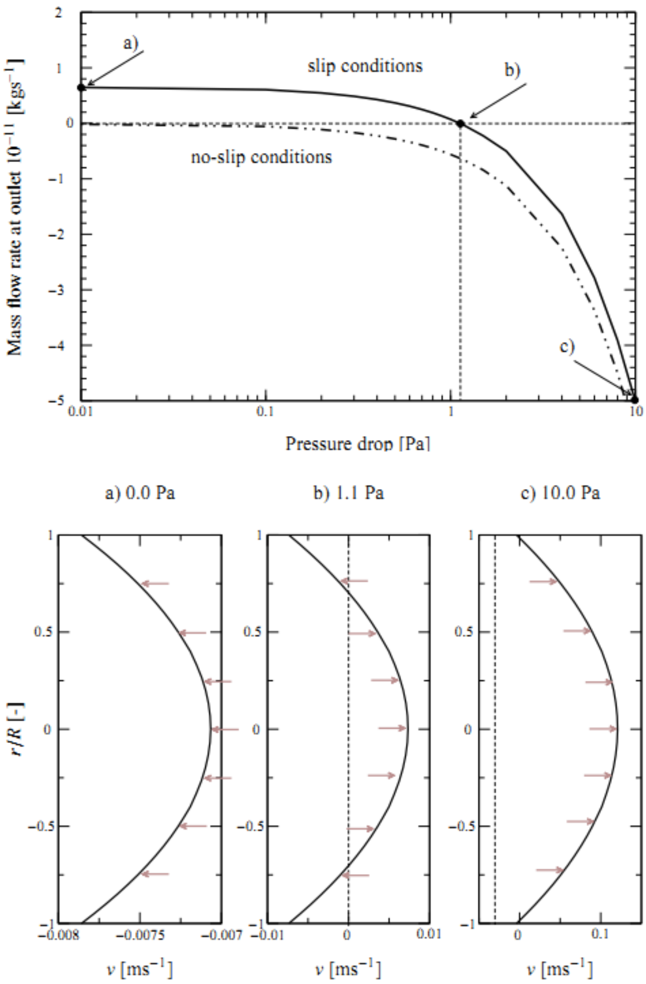

Figure 2. The calculated mass flow rate and relevant velocity profiles in the Maxwell capillary tube for given constant temperature difference $d\theta = 100$ K, and for different dp : a) 0 Pa, b) 1.1 Pa and c) 10 Pa. The case a) describes pure thermal transpiration (no pressure driven flow), where slip velocity $v_s = 0.0077 m/s$ drives the bulk flow of a gas.

CONCLUSION

In the paper the applications of the extended solid-fluid contact equations, including the different surface mobility mechanisms are presented in order to explain the enhanced flow in micro-channels.

Boundary force is a sum of friction and mobility force: $\mathbf{f}_{\partial V} = v(\mathrm{v} - \mathrm{v}_{wall}) + (-c_{s,\omega}\mathrm{grad}_s\varpi - c_{s,\theta}\mathrm{grad}_s\theta - c_{s,c}\mathrm{grad}_s c)$ where $c_{s,\omega}$ - pressure transpiration; $c_{s,\theta}$ - thermal transpiration; $c_{s,c}$ - concentration transpiration.

Generalization of the fluid-solid contact boundary slip layer, formulated in the present paper, supplements the original Navier-Stokes model by additional surface quantities like the surface mass and the surface momentum flux. In the present case the slip velocity $\mathbf{v}_s$ is determined from the solution of the complete balance of momentum (11) written within the layer. Since the stress tensors $\mathbf{p}_A$, $\mathbf{p}_B$ are determined in the bulk and cannot be arbitrarily changed at the boundary, such an approach leads to the separation for those constitutive relations which can be imposed to fulfill the surface balance of momentum. There is still an open place for the modeling of the surface momentum diade $\mathbf{p}_s$ and the surface friction force $\mathbf{f}_{AB}$, where indeed a second gradient of surface velocity can be postulated.

[6] Another objective for analytical study lies in exploring the underlying physics of the so called Knudsen paradox. Explanations of this paradox cannot be given by model of Navier slip layer, and needs more advanced method of modeling, [1; 13]. Let recall, that the Knudsen paradox relates to the presence of a minimum of mass flow rate in a function of the Knudsen number. Thus, the exploration of Knudsen paradox and its full understanding also require a considerations on the limit of continuum approaches. It is fact, that the Knudsen-Gaede flow should be a fundamental benchmark for nano-flows of rarefied gases like the Pouiselle or Couette flow at macro-scale.

[7] See: ([14],Appendix,eq.(81))

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