STIRLING ENGINE AS SIMPLE AS POSSIBLE

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ABSTRACT

Within isothermal analysis of Stirling engine analytical expressions for a pressure as a function of crank angle and for a work are found. The efficiency is numerical calculated for a wide range of temperatures. In contrast to the efficiency of the Carnot cycle, which is monotonically increasing function of the ratio of temperatures of heat baths, the efficiency of the Stirling engine has its maximum. The value of this maximum, as well as the corresponding ratio of heat baths temperatures, depends on a molar heat capacity of the working substance.

INTRODUCTION

In 1816 Stirling brothers patented an engine known as the Stirling engine. Stirling brothers patented five different kinds. Although there are several different kinds of Stirling engines they all have two major things in common. A gas as working substance never leave the engine. This engine uses an external heat source of any kind like, Sun, geothermal water, fossil resources, biomass or nuclear reactor.

BASIC ASSUMPTIONS

In this paper we consider α type of Stirling engine. It consists of two power pistons situated in mutually perpendicular cylinders. Pistons are interconnected via flying wheel as it is shown in Fig. 1.

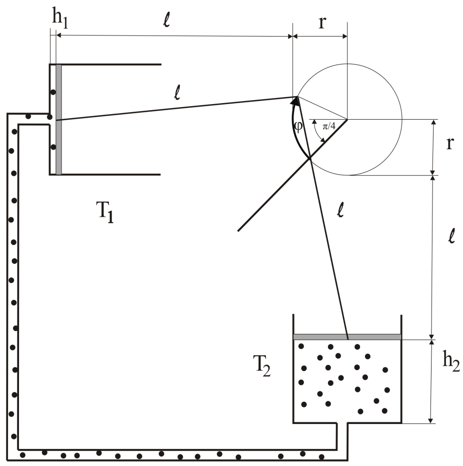

Figure 1. α Stirling engine.

We assume isothermal process between gas and heat baths. This assumption is in common with the Schmidt theory [1; 2] of Stirling engine. The basic difference between Schmidt analysis and one in this paper is the method of calculation of absorbed heat. Schmidt theory assumes that gas absorbs heat in expansion part of the cycle. In contrary to this, rather arbitrary assumption, we keep track of sign of infinitesimal heat exchange between gas and hot cylinder. In this way the regions of heat absorption is clearly determined. This allow us to find the efficiency as a function of ratio of bath temperatures. Parameters is the gas molar heat capacity.

CYCLE

The speed of pressure transmission is equal to the sound speed. Pistons speeds are much less than speed of sound, Neglecting viscous effects pressures in both cylinders are practically equal. For the sake of simplicity the following assumptions are made:

There is no dead volume. Pistons move from the bottom of cylinders.
A lengths of shafts (l) are much longer than radius (r) of circle described by shafts ends, $r/l \ll 1$.
Working substance is an ideal gas.

From the mechanical point of view Stirling engine has one degree of freedom, the crank angle. A branch of crank angle is defined with radius of flying wheel that ends at shafts ends. The coincidence of this radius with the line of symmetry of Stirling engine, depicted in Fig. 1, at minimum value of the total gas volume defines the zero angle.

Taking into account above mentioned assumptions volumes of the gas in cylinders are,

$$V_1 = Ar\left[1-\cos\left(\varphi-\frac{\pi}{4}\right)\right], \tag{1}$$

$$V_2 = Ar\left[1+\sin\left(\varphi+\frac{\pi}{4}\right)\right], \tag{2}$$

where A is the area of the cross section of cylinders. The total volume is

$$V = Ar\left(2-\sqrt{2}\cos\varphi\right). \tag{3}$$

Assuming isothermal processes in cylinders and using ideal gas equation of state we find the dependence of the pressure on the crank angle,

$$p = \frac{nRT_1 B}{Ar\left(1+B-\sqrt{1+B^2}\cos(\varphi-\delta)\right)}. \tag{4}$$

Here n is the total number of moles, $B = T_2/T_1$ where T_1 and T_2 are bath temperatures, and

$$\delta = arctgB - \frac{\pi}{4} \tag{5}$$

is delay angle. It shows how many degrees a maximum of pressure follows after the total volume has achieved its minimum value. In the case of equal bath temperatures these extrema occurs at the same value of the crank angle ($\varphi = 0$), as it should be. The delay angle tends to its maximum value , $\pi/4$, when ratio of bath temperatures goes to infinity.

The cycle of Stirling engine in the (p,V) diagram is depicted in Fig. 2. Note that points in this (p,V) diagram do not represent a state of working substance. Within isothermal approach. gas is in equilibrium state only within cylinders while gas as a whole is in nonequilibrium state.

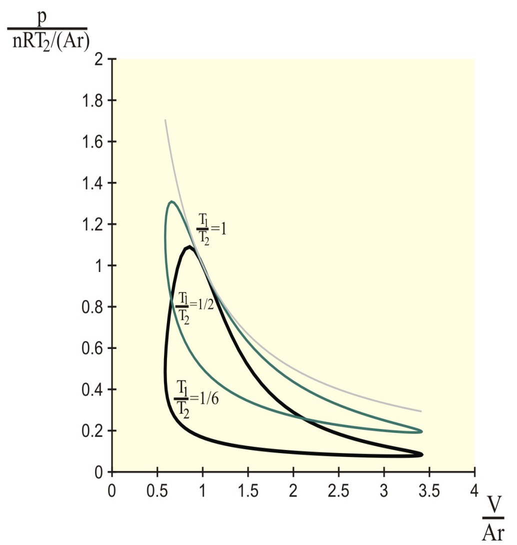

Figure 2. Cycle of Stirling engine for a different values of the ration of bath temperatures.

WORK

Work done by working substance within one cycle is equal to the surface within cycle in (p,V) diagram (Fig. 2),

$$W = \int_0^{2\pi} pdV. \tag{6}$$

Inserting pressure from Eq. (4) into above expression work becomes,

$$W = \sqrt{2}nRT_1\pi(B-1)\sqrt{B}\frac{1+B-\sqrt{2B}}{1+B^2}. \tag{7}$$

The dependence of work on the bath temperatures ratio, B, is shown in the Fig. 3.

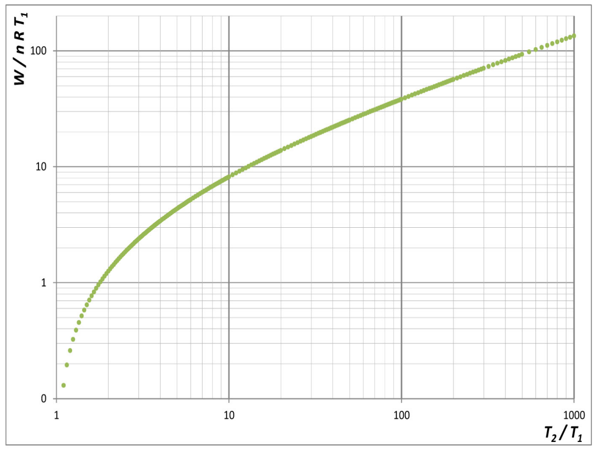

Figure 3. Work done by the Stirling engine as a function of the ratio of bath temperatures.

ABSORBED HEAT

Only the working substance that enters into warm cylinder absorbs heat. Infinitesimal heat exchanged between hot bath and working substance is, according to the first law of thermodynamics,

$$dQ = dn_2 C_V\,(T_2 - T_1) + pdV. \tag{8}$$

In order to find absorbed heat we have to find out the angle interval characterised with $dQ > 0$. Using the principle of mass conservation (fixed number of moles of working substance) and Eq.(3), after lengthily, but otherwise straightforward calculations we get,

$$dQ = BnRT_1\left[\frac{\frac{C_V}{R}(B-1)\left[\cos\left(\varphi-\frac{\pi}{4}\right)-\sin\left(\varphi-\frac{\pi}{4}\right)-1\right]}{\left(1+B-\sin\left(\varphi-\frac{\pi}{4}\right)-B\cos\left(\varphi-\frac{\pi}{4}\right)\right)^2}+\right.$$
$$\left.+\frac{\sqrt{2}\sin\varphi\left\{1+\sin\left(\varphi-\frac{\pi}{4}\right)+B\left[1-\cos\left(\varphi-\frac{\pi}{4}\right)\right]\right\}}{\left(1+B-\sin\left(\varphi-\frac{\pi}{4}\right)-B\cos\left(\varphi-\frac{\pi}{4}\right)\right)^2}\right]d\varphi. \tag{9}$$

Equation $dQ = 0$ defines the region of heat absorption. We have found numerically solutions of following equation,

$$\frac{C_V}{R}(B-1)\left[\cos\left(\varphi-\frac{\pi}{4}\right)-\sin\left(\varphi-\frac{\pi}{4}\right)-1\right]+$$
$$+\sqrt{2}\sin(\varphi)\left\{1+\sin\left(\varphi-\frac{\pi}{4}\right)+B\left[1-\cos\left(\varphi-\frac{\pi}{4}\right)\right]\right\}=0 \tag{10}$$

The results are shown in Fig. 4.

As it has been expected gas absorbs heat during expansion $(0 < \varphi < \pi)$ and releases it during compression for $B = 1$. As ratio of temperatures goes to infinity gas absorbs heat within interval $-\pi/6 < \varphi < \pi/4$.

Absorbed heat is

$$Q^+ = \int_{\varphi_1}^{\varphi_2} dE + \int_{\varphi_1}^{\varphi_2} pdV, \tag{11}$$

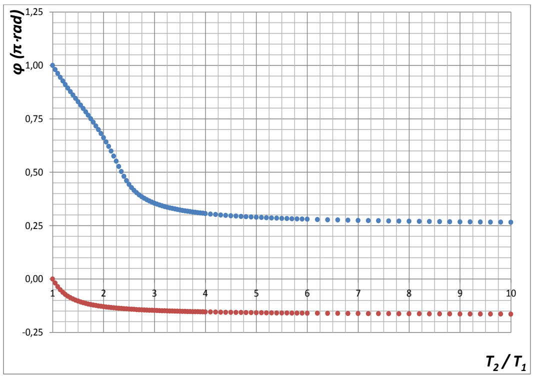

Figure 4. Points of zero absorbed heat φ_1 i φ_2 as a function of ratio of bath temperatures.

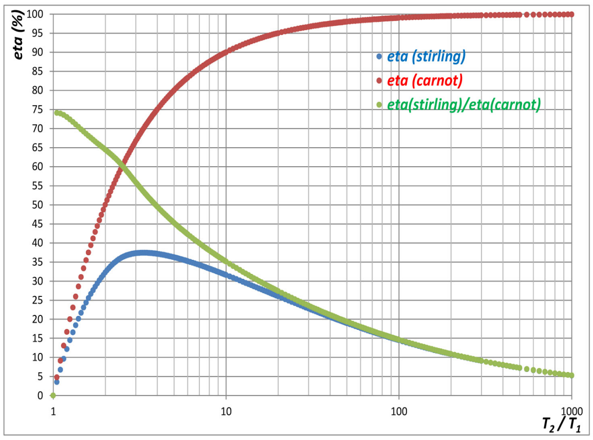

Figure 6. The efficiency of the Stirling engine compared to the efficiency of the Carnot cycle as a function of ratio of temperatures.

where limits of integrations are the zero points of infinitesimal change of heat.

Heat absorbed and released by working substance and work are shown as a function of ratio of bath temperatures in the Fig. 5.

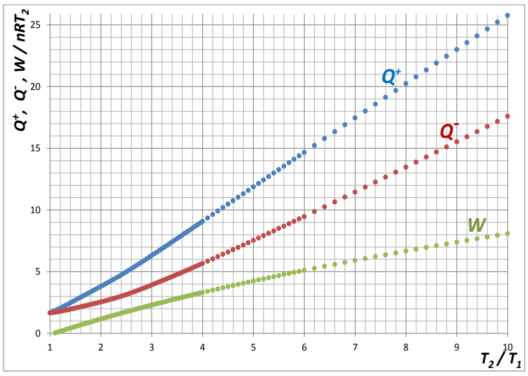

Figure 5. Absorbed, released heat and work as a function of the ratio of bath temperatures.

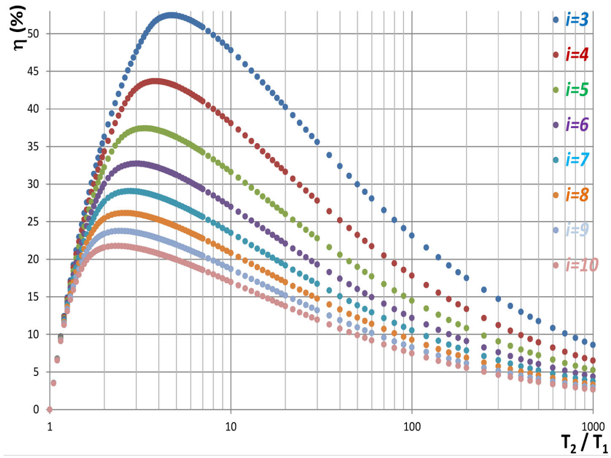

Figure 7. The efficiency of the Stirling engine. A parameter is the molar heat capacity, $C_V = i \cdot R/2$.

EFFICIENCY

The most important parameter of any engine is efficiency. Having calculated absorbed heat and work it is easy to determine efficiency of the Stirling engine. Efficiency is compared with the efficiency of the Carnot cycle as it is shown in Fig. 6.

In contrast to the efficiency of Carnot cycle that is increasing function of the temperature ratio the efficiency of Stirling engine exhibits its maximum. The position of maximum and its value are functions of working gas heat capacity (see Fig. 7). One gets the highest possible efficiency achieves for monoatomic gases at $\approx T_2/T_1 = 3,4$.

DISCUSSION AND CONCLUSION

The efficiency of Stirling engine is calculated within isothermal approach. The basic assumption is isothermal process between gas and heat baths. Most cylinders are closed with disc-like pistons. This design does not ensure isothermal process. An adiabatic process better describes gas state then isothermal ones. In order to ensure absorption of heat one have to add thermal regenerator which is in fact heat exchanger. Heat exchanger introduce dead volume. Dead volume as well as adiabatic process cause the reduction of efficiency. Regenerator keeps a part of working gas that does not exchange heat between hot and cold cylinder. Due to the adiabatic process there is reduced heat exchange between working gas and he baths.

According to analysis exposed in this paper efficiency of the Stirling engine can be even higher than 50% in the case of monoatomic gas as a working substance. The fact that monoatomic gas gives higher efficiency then polyatomic gases is in accordance with kinetic theory of gases. Namely, only translation degrees of freedom contributes to work.

NOMENCLATURE

Symbol	Quantity	SI Unit
l	Length of shaft	m
r	Radius of circle described by shafts ends	m
φ	Crank angle	rad
A	Area of piston basis	m^2
V_1	Volume of working substance in cold cylinder	m^3
V_2	Volume of working substance in hot cylinder	m^3
V	Total Volume of working substance	m^3
p	Pressure	Pa
n	Amount of substance	mol
R	Gas constant	$J/(mol\ K)$
T_1	Temperature of the cold bath	K
T_2	Temperature of the hot bath	K
W	Work	J
Q	Heat	J
E	Internal energy	J
C_V	Molar heat capacity	J/mol
η	Efficiency	

REFERENCES

[1] I. Urieli and D. Berchowitz, *Stirling Cycle Engine Analysis*, Adam Hilger, Bristol, 1984.

[2] J. Škorpk, The Amount of Regenerated Heat Inside the Regenerator of a Stirling Engine, *Acta Polytechnica,* vol. 48, pp. 10-14,2008