MINIMUM FLOW POTENTIAL (MASSIEU FUNCTION) IN FLOW SYSTEMS AND CONNECTION TO ENTROPY PRODUCTION EXTREMA

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EXTENDED ABSTRACT

Recently, the first author presented a new formulation of non-equilibrium thermodynamics, based on Jaynes' maximum entropy (MaxEnt) method, for the analysis of dissipative flow systems [1,2,3,4]. The analysis employs a *flux entropy* concept, representing the uncertainty associated with the set of instantaneous fluxes through the boundary of a fluid control volume, as well as the instantaneous rates of spontaneous chemical reactions within the control volume. Applying MaxEnt, these are constrained by mean values of the fluxes through and rates within the element. For an open system, this yields a new nonequilibrium thermodynamic potential (Massieu function), which can be termed the *flux potential*, which is minimised at steadystate flow. This minimum then reduces, in different circumstances, to a minimum or maximum in the rate of entropy production, suggestive of the respective extremum principles advocated by Prigogine [5] or Paltridge and Zeigler [6, 7]. The implications of the analysis for dissipative flow systems have subsequently been explored [1,2,3,4,8]. The analysis leads naturally to a thermodynamics-inspired mathematical formulation of flow systems, with conjugate extensive and intensive parameters (flows and gradients), first-order and second-order derivatives (giving susceptibilities, fluctuations and Maxwell reciprocal relations), a Legendre transformation between entropy- and potential-based representations, and a Riemannian geometric representation of the manifold of steady states [1,2,3,4,8]. This framework provides a new technique for prediction of the steady state of a flow system, subject only to summary information about the dynamics (e.g. without requiring the full, time-varying Navier-Stokes or energy equations).

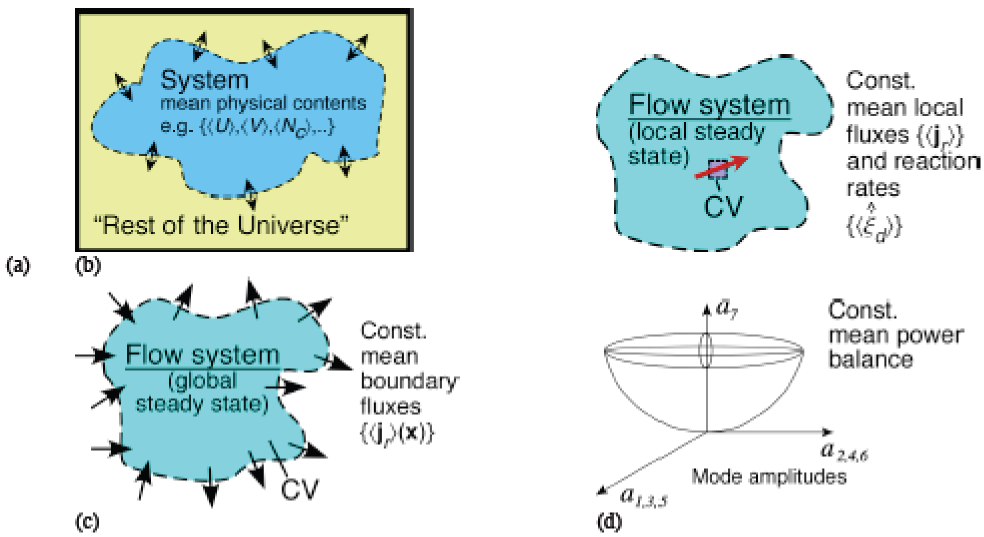

Figure 1: Types of systems amenable to analysis by MaxEnt: (a) equilibrium systems, (b) local and (c) global steady-state flowsystems [1,2,3,4] and (d) Galerkin reduced-order model [9,10,11].

In this study, a generic version of the derivation is first provided, encompassing four seemingly disparate formulations of (a) equilibrium thermodynamics; (b) local and (c) global steady-state flow in physical space; and (d) a Galerkin spectral model, based on a principal orthogonal decomposition of the flow field [9,10,11]. These are represented in Figure 1. The local and global flow system representations require careful control volume analysis [4], and lead into a discussion of scale effects, the definition of steady state, analysis by compartments and the effect of radiative transfer. In the Galerkin decomposition, the MaxEnt closure is applied to a seven-mode Galerkin model of an incompressible periodic cylinder wake at Re=100. The MaxEnt prediction of mean amplitude values is shown to be in close agreement with Direct Navier-Stokes simulations, at much lower computation cost. For all four representations, the choice of prior probabilities is critical to the analysis, and is examined in detail.

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