CONSTRUCTAL LAW: DESIGN AS PHYSICS

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ABSTRACT

A law of physics is a concise statement that summarizes a phenomenon that occurs everywhere in nature. A phenomenon is a fact, circumstance or experience that is apparent to the human senses and can be described. The phenomenon summarized by the constructal law is the occurrence and evolution of designs in nature. The phenomenon is the time direction of the movie of design evolution. The direction is universal, toward configurations and rhythms (designs) that flow and move more easily, for greater access, over time. Based on its record, the constructal law accounts for the design phenomenon and also for all the phenomena associated ad-hoc with final-design (destiny) statements of "optimality" (min, max) that have been proposed. Most notably, the constructal law accounts for the contradictory final-design statements of minimum entropy production and maximum entropy production, and minimum flow resistance and maximum flow resistance. On the earth's surface, the design in nature phenomenon facilitates access for everything that flows, spreads and is collected: river basins, atmospheric and ocean currents, animal life and migration, and our civilization (the evolution of the "human and machine species").

DESIGN AS SCIENCE

Design in nature is the main theme in science today. It began with geometry and mechanics, which are about designs (configurations), their principles, and the contrivances made based on designs and principles. Science has always been about the human urge to make sense out of what we discern: numerous observations that we tend to store compactly as "phenomena" and, later, as much more compact "laws" that account for the phenomena.

To see the position of design in nature as a subject in physics, it is necessary to recall that thermodynamics rests on two laws that are both "first principles". The first law commands the conservation of energy in any system. The second law commands the presence of irreversibility (i.e. the generation of entropy) in any system. The permanence and extreme generality of the two laws are consequences of the fact that in thermodynamics the "any system" is a black box. It is a region of space, or a collection of matter without specified shape and structure. The two laws are global statements about the balance or imbalance of the flows (mass, heat, work) that flow into and out of the black box.

Nature is not made of boxes without configuration. The systems that we identify in nature have shape and structure. They are resoundingly macroscopic, finite size, and recognizable as sharp lines drawn on a different background. They have patterns, maps, rhythms and sounds. The very fact that they have names (river basins, blood vessels, trees) indicates that they have unmistakable appearances.

In my 1997 thermodynamics book [1], I pointed out that the laws of thermodynamics do not account *completely* for the systems of nature, even though scientists have built thermodynamics into thick books in which the two laws are just the introduction. The body of doctrine is devoted to describing, designing and "improving" things that seem to correspond to systems found in nature, or can be used by humans to make life easier. Nowhere is this more evident than in the method of Entropy Generation Minimization [2, 3] where design is recognized as "thermodynamics", even though neither of the two laws accounts for the natural occurrence of "design" and "design evolution" phenomena.

If physics is to account for the systems of nature, then thermodynamics must be strengthened with an additional self-standing law (i.e., with another first principle) that covers all phenomena of design occurrence and evolution. To achieve this, I added to physics the constructal law [1, 4], which states briefly that

> "For a finite-size system to persist in time (to live) its configuration must change such that it provides easier access to its currents" [1, 4].

The constructal law is a definition of life in the broadest possible sense: to be alive, a system must be able to flow and to morph in time so that its currents flow more and more easily. Live are the water streams in the river basins and the streams of animal mass flowing on the landscape, which are better known as animal locomotion and migration. Live are the animate and the inanimate systems that flow, move, and change configuration. The constructal law commands that the changes in configuration must occur in a particular *direction* in time (toward designs that allow currents to flow more easily). The constructal law places the concepts of "design" and "evolution" centrally in physics.

The Constructal Law is a field that is expanding rapidly in physics, biology, technology and social sciences. The field was reviewed in 2006 [5, 6], and now it is expanding even more rapidly. No less than 13 books have been published on the Constructal Law since 2006 [5-19]. In April 2013, the entry "constructal" on ISI revealed an h index of 39 and a total number of 7000 citations. On Google Scholar, the word "constructal" yielded more than 2,300 titles.

DESIGN AND EVOLUTION: ANIMATE AND INANIMATE

The constructal law of design in nature constitutes a unified view of design evolution. It predicts evolution in all the domains in which evolutionary phenomena are observed, recorded and studied scientifically: animal design, river basins, turbulent flow, dendritic crystals, animal movement, athletics, technology evolution and global design. Some of the most common animate and inanimate systems that we predicted with the constructal law are sketched in Fig. 1.

Evolution means design modifications, in time. How these changes are happening are *mechanisms*, and *mechanism* should not be confused with *law*. In the evolution of biological design, the mechanism is mutations, biological selection and survival. In geophysical design, the mechanism is soil erosion, rock dynamics, water-vegetation interaction, and wind drag. In sports evolution, the mechanism is training, recruitment, mentoring, selection, and rewards. In technology evolution, the mechanism is liberty, freedom to question, innovation, education, trade, theft and emigration.

What flows through a design that evolves is not nearly as special in physics as how the flow system generates its configuration in time. The "how" is the physics principle—the constructal law. The "what" are the mechanisms, and they are as diverse as the flow systems themselves. The "what" are many, and the "how" is one.

Having "impact" on the environment is synonymous with having design in nature. To flow means to get the surroundings out of the way. There is no part of nature that does not resist the flows and movements that attempt to get through it. Movement means penetration, and its name differs depending on the direction from which the phenomenon is observed. To the observer of river basins, the phenomenon is the emergence and evolution of the dendritic vasculature. To the observer of the landscape, the phenomenon is erosion and the reshaping of the earth's crust.

This mental viewing of design generation and environmental impact as a unitary design in nature is universally applicable. Think of the paths of animals, versus the river-like paths and burrows dug into the ground. Think of the migration of elephants, versus the toppling of trees. The patterns of social dynamics go hand-in-glove with impact on the environment.

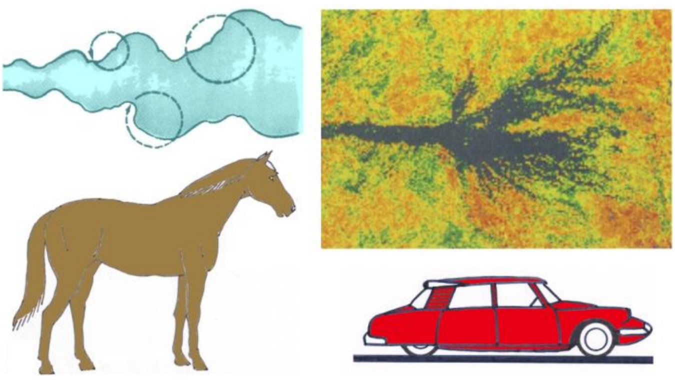

Figure 1 The larger are more efficient, faster, live longer and travel farther lifetime: vehicles, animals, rivers and the winds.

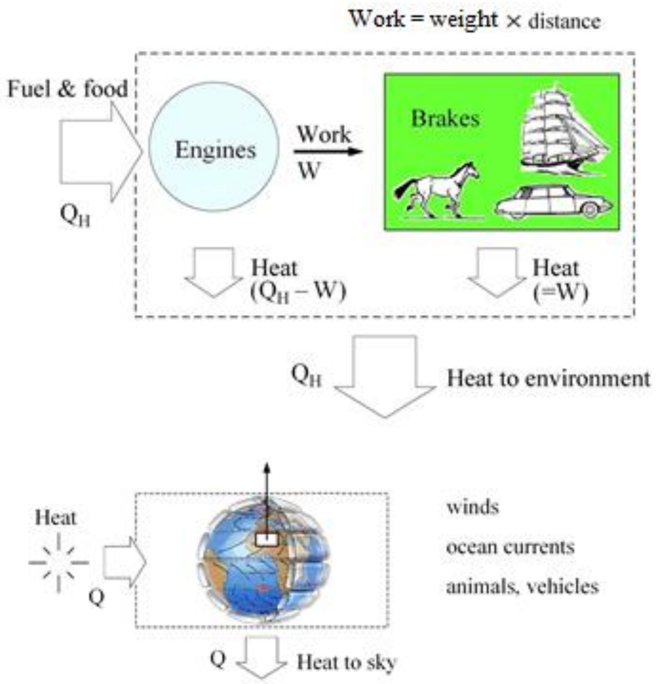

Figure 2 Everything that moves on earth is driven. It moves because an engine dissipates its work output into a brake.

Animal locomotion is "guided locomotion", with design—it is efficient, economical, safe, fast and purposefully straight. This is the constructal design of animal and human locomotion, and it is the complete opposite of Brownian motion. The constructal design of animal locomotion is much more complicated and perfected than the thermodynamics of balancing two work efforts, one on the vertical (lifting weight) and the other on the horizontal (getting the environment out of the way), which led to the discovery of the allometric relation between all animal speeds, body frequencies and body mass [20-24].

The movement of the body weight on alternating legs is equivalent to the view of walking and running as falling-forward locomotion. The legs are the two spokes of the human wheel [25] from which the other spokes are missing, and which make the animal wheel the lightest wheel.

The constructal design of all urban movement is such that, at all length scales, *the time needed to travel short and slow is roughly the same as the time needed to travel long and fast*. The need to "travel short and long" to move on a territory (area, volume) was the example with which the constructal theory of design in nature began in 1996 [4]. This continued with explaining why the design of the Atlanta airport is efficient, and why the designs of new airports are evolving toward the Atlanta design [26, 27].

In the Atlanta design, the short and slow is walking along the concourse, and the long and fast is riding on the train. In the city design, at the smallest scale the time balance is between walking from the house to the car and riding on the small street. At the next scale, the balance is between riding on streets (short, slow) and avenues (long, fast), and so on to larger scales: avenues and highways, highways and intercity train and air travel, short flights and long flights, all the way to the scale of the globe. We have applied this principle to the design of the infrastructure (inhabited spaces) for fastest and safest evacuation of pedestrians, from crowded areas and volumes [28, 29].

The slow and short are many, and the fast and long are few. The design of all movement on earth, animate and inanimate (river basins, eddies of turbulence, animal life, trucks on the roads, airplanes in the air, streets in the city) is one design: *few large and many small* [20, 30].

The effect of life is *measurable* in terms of the mass moved over distances during the life time of the flow system (Fig. 2). The work required to move any mass on earth (vehicle, river water, animal mass) scales as the weight of that mass time the distance to which it is moved horizontally, on the landscape. It is this way with the life of the river basin and the animal, and it is the same with the life of man, family, country and empire. The economic activity of a country is all this movement—mass (people, goods) moved to distances. Because every movement is proportional to the amount of fuel burned in order to drive it, the entire economic activity on a territory must be proportional to the amount of fuel consumed on that territory. This view predicts that the annual GDP of a country should be proportional to the amount of fuel burned in the country (i.e. the useful energy generated and destroyed) [31]. This is confirmed by the economics data plotted in Fig. 3.

Animals have been spreading in space, in this unmistakable time direction dictated by the constructal law: from sea to land, and later from land to air [32]. The movement of the human & machine species evolved in the same direction, from small boats with oars on rivers and along the sea shore, to the wheel and vehicles on land, and most recently to aircraft.

The same movie (because this is what the occurrence and evolution of design is, a time sequence of images) shows that speeds have been increasing in time, and will continue to increase. For the same body mass, the runners are faster than the swimmers, and the fliers are faster than the runners. This movie is the same as the evolution of inanimate mass flows, for example, the river basins. Under the persisting rain, all the channels morph constantly, to flow more easily.

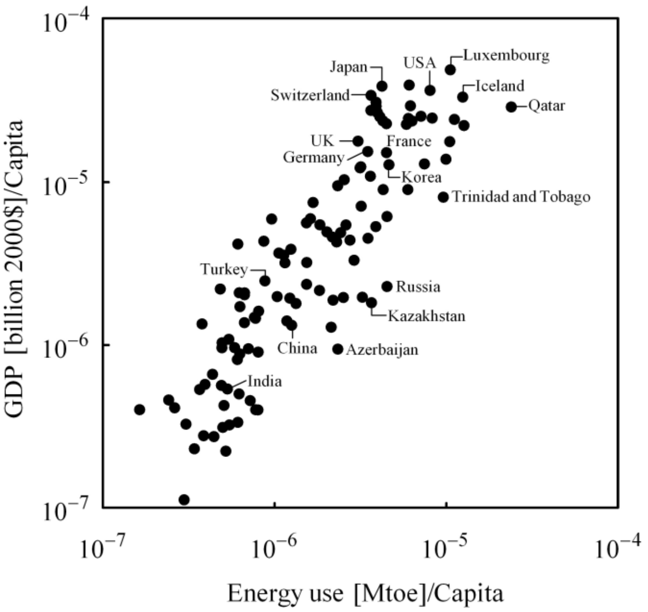

Figure 3 Economic activity means movement, which comes from the burning of fuel for human needs. This is demonstrated by the annual GDP of countries all over the globe, which is proportional to the fuel burned in those countries (data from International Energy Agency. Key World Energy Statistics, 2006). In time, all the countries are moving up and to the right, on the bisector.

Spreading and collecting flows occupy areas and volumes that have S-shaped history curves predicted with the constructal law [33-35]. Design is the speed governor of nature. None of the changes observed in politics, history, sociology, animal speed and river speed are spinning out of control. None of the expansions feared in geography, economics and urbanism are slamming into a brick wall.

CONSTRUCTAL LAW VERSUS FINAL DESIGN

The constructal law is not a statement of optimization, maximization, minimization, or any other mental image of "final design" or "destiny". The constructal law is about the direction of evolution in time, and the fact that design in nature is not static: it is dynamic, ever changing, like the images in a movie at the cinema. This is what design and evolution are in nature, and the constructal law captures them completely. Evolution never ends.

There have been many proposals of final-design in science, but each addresses a narrow domain, and, as a consequence, the body of optimality statements that have emerged is self-contradictory, and the claim that each is a general principle is easy to refute. Here are the best known statements:

(i) Minimum entropy generation and maximum efficiency are used commonly in engineering.
(ii) Maximum entropy generation is being invoked in geophysics.
(iii) Maximum "fitness" and "adaptability" (robustness, resilience) are used in biology.
(iv) Minimum flow resistance (fluid flow, heat transfer, mass transfer) is invoked in engineering, river mechanics and physiology.
(v) Maximum flow resistance is used regularly in physiology and engineering, e.g. maximum resistance to loss of body heat through animal hair and fur, or through the insulation of power and refrigeration plants, the minimization of fluid leaks through the walls of ducts, etc.
(vi) Minimum travel time is used in urban design, traffic, transportation.
(vii) Minimum effort and cost is a core idea in social dynamics and animal design.
(viii) Maximum profit and utility is used in economics.
(ix) Maximum territory is used for rationalizing the spreading of living species, deltas in the desert, and empires.
(x) Uniform distribution of maximum stresses is used as an "axiom" in rationalizing the design of botanical trees and animal bones.
(xi) Maximum growth rate of flow disturbances (deformations) is invoked in the study of fluid flow disturbances and turbulence.
(xii) Maximum power was proposed in biology and is used in physics and engineering.

This list is incomplete. Even though the optimality statements are contradictory, local, and disunited on the map of design in nature, they demonstrate that the interest in placing design phenomena deterministically in science is old, broad and thriving. One example is flow of stresses phenomenon [36] that accounts for the emergence of solid shape and structure in vegetation, skeleton design, and technology. The flow of stresses is an integral part of the design-generation phenomenon of moving mass more and more easily on the landscape [11, 30].

Another example is the contradiction between minimum and maximum of entropy generation [see (i) and (ii) above], which was resolved based on the constructal law in 2006 [26]. The flowing nature is composed of systems that move as engines connected to brakes. In time, the "engines" of nature acquire configurations that flow more easily, and this means that they evolve toward less entropy generation, and more production of motive power per unit of useful energy (exergy) used. At the same time the "brakes" of nature destroy the produced power, and this translates into their evolution toward configurations that dissipate more and more power. The principle is not the maximum or the minimum, or the fact that the "engine + brake" constitution of nature (Fig. 2) brings them together. The principle is the design evolution of "engine" configurations in the time direction dictated by the constructal law and the design evolution of "brake" configurations in the same direction over time.

To think that design evolution means "evolution toward patterns of least resistance" is, at best, a metaphor. What "resistance" when walking in total freedom alone on the beach? What "resistance" when sitting down on the train in the Atlanta airport, and wanting to arrive at your gate faster? What "resistance" when searching for a cheaper ticket between Atlanta and Hong Kong? What "resistance" when the lucky animal finds food and we find oil? What "resistance" when the snowflake grows freely as a daisy wheel of trees? Furthermore, what is "least" (or maximum, minimum) about any design? Who is to know that the urge to have an even better design has reached the end? Who is the to know that a final design exists?

Resistance is a concept from electricity (voltage divided by current), which was introduced subsequently in fluid mechanics (pressure difference divided by mass flow rate) and heat transfer (temperature difference divided by heat current). In pedestrian and animal movement the current is obvious: it is the flow rate of human mass through a plane perpendicular to the flow path. Not obvious is the "difference" (voltage, pressure, temperature) that drives the pedestrian flow.

I faced these questions squarely when I composed the constructal law in 1996 [1, 4], and this is why I summarized the design-in-nature phenomenon with a statement of all physics that is universally applicable, without words and such as resistance and static end-design (optimum, min, max), cf. Section 1. Yet, in our morphing movement (i.e., life) on earth, we rely on thoughts such as greater access, more freedom, go with the flow, shorter path, less resistance, longer life, less expensive and greater wealth. These ideas guide us, like the innate urges to have comfort, beauty and pleasure.

The constructal law empowers the mind to fast-forward the design evolution process. This is in fact what the human mind does with any law of physics—the mind uses the law to predict features of future phenomena. Knowing ahead is also an expression of the constructal law [32], because all animal design is about moving more and more easily on the landscape, and this includes the phenomenon of cognition – the urge to get smarter, understand and remember faster, so that the animal can get going and place itself out of danger. Relying on the constructal-law direction to fast-forward the design is useful.

CONSTRUCTAL LAW VERSUS SECOND LAW

The constructal law, the first law, and the second law are first principles. The constructal law is a useful reminder of not only what is missing in thermodynamics (the design & evolution principle) but also of what is present.

For example, we often read that the second law states that "entropy must increase", and that the "classical" laws of thermodynamics pertain to "equilibrium states". Many even teach that thermodynamics should be called thermo"statics". Such statements are unrecognizable from a point of view rooted in thermodynamics. Here is the correct statement of the second law, made by two of its three original proponents in 1851-1852 (the other was Rankine) [26]:

Clausius: No process is possible whose sole result is the transfer of heat from a body of lower temperature to a body of higher temperature.

Kelvin: Spontaneously, heat cannot flow from cold regions to hot regions without external work being performed on the system.

Note that the second law says absolutely nothing about "equilibrium states", "entropy", "classical", and "statics".

Like any other law of physics, the second law of thermodynamics is a concise summary of observed facts. A law unifies phenomena (the observed facts). The second-law phenomenon is irreversibility. The correct summary of the phenomenon of irreversibility is due to Clausius and Kelvin above, and to others who made demonstrably *equivalent* statements (for a review, see Ref. [1]). The only relevant question about the second law statement is whether it is correct. The evidence is massively in support of answering "yes", based on all the machines that have been built by engineers successfully because they relied on the second law of thermodynamics of Rankine, Clausius, and Kelvin. These machines are every day futuristic (not "classical"), they are full of life and motion (not in "equilibrium"), and are dynamic (not "static").

The constructal-law phenomenon is the occurrence of design and evolution in nature. The constructal law recognizes the natural tendency of evolution toward "easier access in time". The word "access" means the opportunity to enter and move through a confined space such as a crowded room. This mental viewing covers all the flow design and evolution phenomena, animate and inanimate, because they all morph to enter and to flow better, more easily, while the flow space is constrained. This is why "finite-size" is mentioned in the statement of the constructal law (Section 1). See also the comments on flow resistance, at the end of Section 3.

If the reader has a particular flow system in mind, say, air flow in lungs or electricity in lightning, then the reader can express the evolutionary design toward easier access in terms of locally appropriate variables and units. Yet, the fluid flow terminology of the lungs has no place in the analysis of the flow of electricity as a lightning tree, and vice versa. What is the same in both examples is the first principle: the evolution of design toward easier access, through changes in flow configuration in a finite-size system.

CONSTRUCTAL THERMODYNAMICS: PHYSICS AND BIOLOGY

The constructal law is universally valid, as physics, precisely because it is not a statement of optimality and final design [all the optimization statements have failed: see again (i) – (xii) in Section 1]. A new law does not have to be stated in mathematical terms (e.g., thermodynamic variables, units). For example, the second law of thermodynamics was stated in words, as a mental viewing, not as a mathematical formula (see the Clausius and Kelvin statements). The mathematization of the second law statement (and of thermodynamics) came later. The same evolution occurred in constructal theory. The 1996 statement of the constructal law was followed in 2004 by a complete mathematical formulation of *constructal-law thermodynamics* [37], Fig. 4.

The constructal law is a contribution to physics and evolutionary biology because it simplifies and clarifies the terminology that is in use, and because it unifies it with the biology-inspired terminology that is in use in many other fields such as geophysics, economics, technology, education and science, books and libraries [38]. This unifying power is both useful and potentially controversial because it runs against current dogma.

For example, the constructal designs of the river basin, the tree distribution in the forest, the animal distribution and "animal flow" on the landscape, and all the other "few large and many small" designs such as the food chain, demography and transportation are viewed as *whole* architectures in which what matters is the better and better flow over the global system. In all such architectures, the few large and many small flow together. They collaborate, adjust, and collaborate again toward a better flowing whole, which is better for each subsystem of the whole. This holistic view of design phenomena represents two new steps:

First, the concept of "better" is defined in physics terms, along with direction, design and evolution (cf. the constructal law). In biology, this step unveils the concept of random events and mutations ("changes", from this to that, from here to there) as a *mechanism* akin to river bed erosion, periodic food scarcity, plagues, scientific discovery, etc., which make possible running sequences of changes that are recognized widely as evolution. This step places in physics the biology terms of natural selection, freedom to change and adapt, survival, and the idea that *there are* better designs.

Second, the constructal view of design and evolution runs against the negative tone of biology-inspired terms that have invaded the scientific landscape, for example, winners and losers, zero sum game, competition, hierarchy, food chain, limits to growth, etc. No, in the big picture, the few large and many small evolve together, in order to survive and to be able to move more mass on the landscape together. The few large do not and cannot eliminate the many small. Their balanced multiscale design gets better and better, for the benefit of the whole flowing system. Contrary to this apparent conflict with standard interpretations of evolutionary biology, what is "good" in biology is good in constructal theory and all the domains of design science that the constructal law covers.

The constructal law is predictive, not descriptive. This is the big difference between the constructal law and other views of design in nature. Previous attempts to explain design in nature are based on empiricism: observing first, and explaining after. They are backward looking, static, descriptive and at best explanatory. They are not predictive theories even though some are called "theory", e.g., complexity theory, network theory, chaos theory, power laws (allometric scaling rules), "general models", and optimality statements (minimum, maximum, optimum).

With the constructal law, complexity and scaling rules are discovered, not observed. Complexity is finite (modest), and is part of the description of the constructal design that emerges. If the flows are between points and areas or volumes, the constructal designs that are discovered are tree-shaped networks. The "networks" are discovered, not observed, and not postulated. Networks, scaling rules and complexity are part of the description of the world of constructal design that emerges predictively from the constructal law.

Constructal "theory" is not the same as constructal "law". Constructal *theory* is the view that the constructal *law* is correct and reliable in a predictive sense in a particular flow system. For example, reliance on the constructal law to predict the evolving architecture of the snowflake is the constructal theory of rapid solidification. Using the constructal law to predict the architecture of the lung and the rhythm of inhaling and exhaling is the constructal theory of respiration.

The law is one, and the theories are many—as many as the phenomena that the thinker wishes to predict by invoking the law.

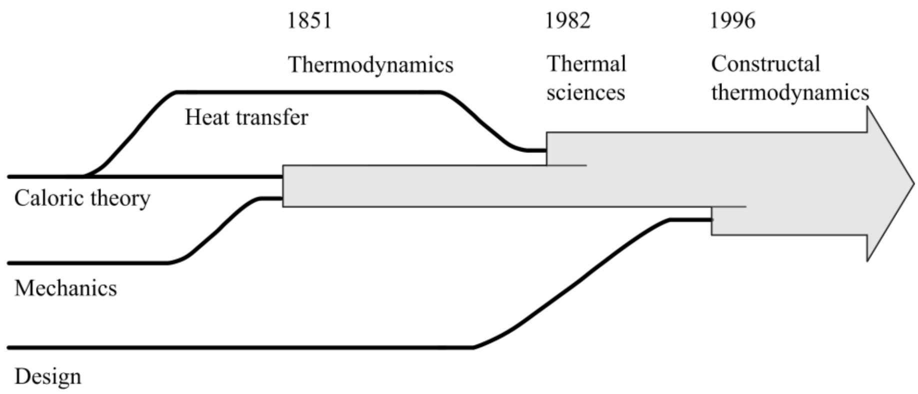

Figure 4 The evolution and spreading of thermodynamics during the past two centuries (after Ref. 2, Diagram 1, p. viii).

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