ON THE STRUCTURE OF ROTATIONAL THERMODYNAMICS FLUXES IN SOLIDS

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EXTENDED ABSTRACT

Linear irreversible thermodynamics is not sufficient to deal with inelastic behaviors of solids and nonlinear thermodynamic theories are then required. Among various existing nonlinear thermodynamic framework for solids, the thermodynamic fluxes are generally assumed to be potential or irrotational. In the well-established multiscale nonlinear thermodynamic framework with internal variables by Rice [1], a flow potential function for the thermodynamic flux in a solid is ensured by Rice's condition that each rate of microscale internal variables be fully determine by its thermodynamic conjugate force. A similar viewpoint was also suggested by Ziegler [3] that all constitutive equations of solids can be derived from two potential functions, the free energy function and dissipation function.

We have shown that the Rice condition is embodied by the nonlinear Onsager reciprocal relations by Edelen [2]. In other words, the Rice condition is a sufficient condition to the nonlinear Onsager reciprocal relations. Taking the dissipation function as the potential function, the Ziegler fluxes are irrotational fluxes obeying the principle of maximum dissipation [3]. As shown by us within Rice's framework, the Ziegler fluxes correspond to the Rice fluxes in which each rate of microscale internal variables is the homogeneous function of its thermodynamic conjugate force in the same degree.

Although the nonlinear thermodynamics based on irrotational assumption has achieved great successes in constitutive description of inelastic behaviours of solids, it is still not sufficient to describe some special phenomenon, e.g., non-associativity, softening and deformation localization, etc., especially for geomaterials. In general, irrotational thermodynamic fluxes exist around thermodynamic equilibrium points, and the irrotational assumption doesn't hold when a thermodynamic state is far from equilibrium states. In order to describe the special phenomenon, rotational thermodynamic fluxes and their inherent structures should be addressed. In other words, the nonlinear thermodynamic mechanisms beyond the potential area should be addressed.

For a rotational thermodynamics flux， the kinetics of numerous microscale internal variables, cannot be derived from a single potential function. It is shown in this paper that the kinetics can be constructed with a number of potential functions based on Darboux theorem [2]. For irrotational fluxes, the principle of maximum dissipation leads to the orthogonality condition by Ziegler [3]. For rotational fluxes with multiple potential functions, several issues are addressed in this paper. Whether or not does the the principle of maximum dissipation still hold beyond the potential area? Whether or not does certain generalized orthogonality condition exist for rotational fluxes with multiple potential functions?

Time-independent inelastic behaviours is also addressed in this paper. It is shown within the framework of Rice that the time-independent inelastic behaviours can be fully determined by the Hessian matrix at the non-degenerate critical point of the free energy function. If the free energy is a direct function of the internal variables, the thermodynamic equilibrium condition leads to associated flow rules. If the free energy is not a direct function of the internal variables, the thermodynamic equilibrium condition could degenerate into several possible thermodynamic equilibrium mechanisms and finally leads to non-associated flow rules.

REFERENCES

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